NASA Technical Memorandum 101564

ACOUSTIC RESPONSE OF A RECTANGULAR WAVEGUIDE WITH A STRONG TRANSVERSE TEMPERATURE GRADIENT

William E. Zorumski

(NASA-TM-101564) ACOUSTIC RESPONSE OF A RECTANGULAR WAVEGUIDE WITH A STRONG TRANSVERSE TEMPERATURE GRADIENT (NASA. Langley Research Center) 35 p CSCL 20A G3/71

N89-24140

Unclas 0210324

March 1989

National Aeronautics and Space Administration

Langley Research Center
Hampton, Virginia 23665-5225

INTRODUCTION

The effect of temperature in acoustics is usually analyzed by allowing the speed of sound to be a function of spatial position[1,2]. This type of analysis ignores the explicit effect of the temperature gradient. It is known that the acoustic perturbation equations contain explicit derivatives of the thermodynamic state variables, such as pressure and density, but these derivatives may often be ignored because they are small on the scale of an acoustic wavelength. Explicit gradient effects have been included in studies of sound propagation in the atmosphere[3,4] where long wavelenghs are of interest. These wavelengths are called infrasonic, where the temporal frequencies are on the order of 1 Hz and less. The temperature gradient in the atmosphere is typically on the order of several kelvins per kilometer so that the wavelength must be long in order for the temperature to vary significantly over a wavelength. Gradient effects may be neglected in studies of audio-frequency (20Hz-20kHz) sound in the atmosphere.

The purpose of this paper to is analyze problems where the temperature gradient is strong and cannot be neglected. One problem considered is a simple rectangular cavity with the temperature being a function of only one spatial coordinate, the one representing the height of the cavity. This situation may occur, for example, within the structures of high-speed aircraft where the skin is heated by a supersonic or hypersonic boundary layer. The cavity could be the interior of a Thermal Protection System (TPS)[5] structural panel such as shown in Fig. 1. The outer skin can be up to 1350 kelvins while the inner skin should be relatively cool, say 300 to 400 kelvins. The temperature gradient within such a panel would then be 10 to 20 kelvins/cm, or six orders of magnitude larger than the typical atmospheric gradient. More importantly, it is large for wavelengths of interest in the analysis of the structural panels. The resonant frequencies of the cavity are important parameters in the dynamic response of a panel over the cavity. If the temperature gradient significantly affects these resonant frequencies, then the gradient is important in predicting the panel response. This paper will define the effect of a temperature gradient on the resonant frequencies of the cavity.

The panel shown in Fig. 1 is filled with insulating blankets. The acoustic properties

of such blankets may be described by the Attenborough[6] model in the case of moderate temperatures, but the inclusion of these properties would complicate the present analysis by introducing the fibrous materials' parameters. Also, the materials would reduce the resonant effects studied here by way of increasing the damping of the acoustic waves. A simpler model, shown in Fig. 2, is used here in order to focus on a single effect—the temperature gradient. The cavity is assumed to have perfectly rigid walls with the temperature varying linearly between the bottom and the top. A perfect gas at constant pressure fills the cavity so that the density varies inversely with temperature.

The essential features of the resonant cavity response may be verified by an experiment on a waveguide. The waveguide, shown in Fig. 3, is like the cavity except that the width is equal to or less than the height. The restricted width eliminates cross modes which depend on the spatial dimension y and thus simplifies the analysis by restricting it to the x-z plane.

A second problem where the strong gradient analysis has application is the response of a recessed pressure sensor. Sensors are installed in recesses when the wall temperature in high-speed boundary layer flows are above the operational limits of the pressure sensors. When this technique is used, a transfer function must be developed to relate the sensed pressure fluctuations to the wall pressure fluctuations. Tijdeman[7,8], has given the transfer functions in recesses where the temperature is constant. These transfer functions show resonant peaks which depend on the cavity depth. The analysis here may be used to show the effect of a strong temperature gradient on the transfer function for a recessed sensor.

The first section of the paper is devoted to developing the acoustic wave equation with a temperature gradient effect. More general equations have been developed than the one used here. Pridmore-Brown[3] has given the perturbation equations including pressure, density, and velocity gradients. Grossard and Hooke[4] give completely general equations in a rotating coordinate system to represent the earth's atmosphere. The specialization here is intended to emphasize a single effect. Consequently, the pressure is assumed to be constant and there is no flow. These simplifications produce a wave equation which is similar to the Helmholtz equation used in elementary acoustics. The equation is generalized, however, by the inclusion of the temperature gradient in a coefficient. This generalized equation,

called the strong gradient equation, shows why the resonant frequencies of a cavity will be altered by the gradient.

The second section of the paper is devoted to the response of a simple rectangular cavity to a monopole source. A set of orthonormal basis functions is constructed such that the gradient-dependent boundary conditions are satisfied. Asymptotic formulas are derived for the effect of the temperature gradients on the modal wave numbers and phase. The Galerkin[10] method is used to express the cavity response in terms of these modes. All integrals involved in the Galerkin procedure are evaluated by exact formulas[1]. Second-order accurate asymptotic formulas are developed for the natural frequencies, modes, and forced response of the cavity.

An experiment is suggested in the third section to verify the effects predicted here. The experiment would use a rectangular waveguide which is simpler than the rectangular cavity. The width of the waveguide is selected to be small in comparison to the height so that fewer modes will propagate. The length of the waveguide is large in comparison to the height. Standing waves are excited by sources at one end of the waveguide and monitored by microphones at the other end. Temperature and temperature gradient are controlled by heated and cooled conducting plates forming the top and bottom walls, respectively, of the waveguide. The other sides are near-perfect insulators so that temperature variation is one-dimensional in accordance with the theory. Excitation frequency is varied with excitation voltage held constant so that the microphone output represents the frequency response function of the waveguide. Comparison of the response functions for different combinations of plate temperature shows the temperature gradient effects. Predictions of the resonant peaks are made to simulate the proposed experiment.

SYMBOLS

a_l	mode normalization constant
$A_{ll'}$	array of weak gradient integrals
$B_{ll'}$	array of strong gradient integrals
$C_{ll'}$	Array of constants
c	sound speed, m/s
$c_{\bar{T}}$	sound speed at average temperature, m/s
c_p	specific heat at constant pressure, $J/Kg \cdot K$
f	frequency, Hz
g	mode function
H	height of rectangular cavity, m
k	unit vector in z-direction
k	wave number, m^{-1}
L	length of rectangular cavity, m
p	pressure, Pa
Q	source strength, s^{-1}
T	temperature, K
$\bar{T}$	average temperature, K
t	time, s
u	velocity, m/s
W	width of rectangular cavity, m
x, y, z	cartesian coordinates
Greek	
α	frequency-shift constant
β	acoustic admittance
γ	ratio of specific heats
δ	Kronecker delta function
ϵ	dimensionless gradient parameter, $\Delta T/\bar{T}$
θ	phase angle, rad

κ	thermal conductivity, $W/m \cdot K$
ξ	dimensionless coordinate, z/H
ρ	density, kg/m^3
τ	thermal gradient
Φ	modified acoustic potential $\sqrt{\frac{T}{T_0}}\Psi$
Ψ	acoustic potential
ω	circular frequency, rad/s

Subscripts:

l	mode shape in z direction
m	mode shape in y direction
n	mode shape in x direction
0	origin
p	constant pressure
T	constant temperature
x	x component
y	y component
z	z component
τ	thermal gradient

Special Symbols:

$\tilde{f}$	acoustic quantity f in time domain
$\hat{f}$	acoustic quantity f in frequency domain
$\bar{f}$	average f, or based on the average f
f'	derivative of f with respect to it's argument

STRONG TEMPERATURE GRADIENT EFFECTS

Acoustic Equations

The following analysis is based on the essentially-standard equations of fluid mechanics, namely the continuity, momentum and energy equations. The continuity equation is taken to be

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{U} = \rho\tilde{Q} \tag{1}$$

The source term on the right of the continuity equation represents a fluctuating volumetric source distribution of strength $\tilde{Q}$, which is small in some sense, representing the magnitude of all acoustic terms. The momentum equation is written without viscous effects included.

$$\frac{D\mathbf{U}}{Dt} + \frac{\nabla p}{\rho} = \mathbf{0} \tag{2}$$

The energy equation is written with heat conduction effects, but without viscous effects. The heat transfer term will be neglected in the acoustic equations on the grounds that the thermal diffusivity is small relative to a typical wqavelength and period of the phenomenon. The energy equation is

$$\frac{Dp}{Dt} = \gamma\frac{p}{\rho}\frac{D\rho}{Dt} + (\gamma - 1)\kappa\nabla^2 T \tag{3}$$

In a system without flow, the velocity is decomposed into a null vector and an acoustic vector.

$$\mathbf{U} \leftarrow \mathbf{0} + \tilde{\mathbf{u}} \tag{4}$$

The steady momentum equation then requires the pressure gradient to be zero.

$$\frac{\nabla p}{\rho} = \mathbf{0} \tag{5}$$

The steady energy equation requires that the temperature satisfy the Laplacian equation.

$$\nabla^2 T = 0 \tag{6}$$

The density is decomposed into steady and fluctuating parts to form the acoustic continuity equation.

$$\rho \leftarrow \rho + \tilde{\rho} \tag{7}$$

The acoustic part of the continuity equation becomes

$$\frac{\partial \tilde{\rho}}{\partial t} + \tilde{\mathbf{u}}\cdot\nabla\rho + \rho\nabla\cdot\tilde{\mathbf{u}} = \rho\tilde{Q} \tag{8}$$

Similarly, the acoustic part of the energy equation is

$$\rho\frac{\partial \tilde{p}}{\partial t} = \gamma p\left(\frac{\partial \tilde{\rho}}{\partial t} + \tilde{\mathbf{u}}\cdot\nabla\rho\right) \tag{9}$$

where the heat transfer term $\kappa\nabla\tilde{T}$ has been neglected. The energy and continuity equations combined relate the fluctuating pressure to the fluctuating velocity and the acoustic source.

$$\nabla\cdot\tilde{\mathbf{u}} + \frac{1}{\gamma p}\frac{\partial \tilde{p}}{\partial t} = \tilde{Q} \tag{10}$$

If an acoustic potential is defined by

$$\tilde{p} = \rho\frac{\partial \tilde{\Psi}}{\partial t} \tag{11}$$

then the continuity-energy (10) equation becomes

$$\nabla\cdot\tilde{\mathbf{u}} + \frac{1}{c^2}\frac{\partial^2 \tilde{\Psi}}{\partial t^2} = \tilde{Q} \tag{12}$$

The acoustic part of momentum equation is

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \frac{\nabla \tilde{p}}{\rho} = 0 \tag{13}$$

and the acoustic velocity in terms of the potential as

$$\frac{\partial^2 \tilde{\mathbf{u}}}{\partial t^2} = -\nabla\frac{\partial^2 \tilde{\Psi}}{\partial t^2} + \frac{\nabla T}{T}\frac{\partial^2 \tilde{\Psi}}{\partial t^2} \tag{14}$$

where the gas law, $p = \rho RT$, has been used to eliminate density in favor of temperature. The acoustic wave equation is formed by solving the momentum equation for $\tilde{\mathbf{u}}$ and substituting this solution into the continuity-energy equation (12).

Harmonic solutions of the form

$$\tilde{\mathbf{u}} = \hat{\mathbf{u}}e^{-i\omega t} \tag{15}$$

$$\tilde{\Psi} = \hat{\Psi} e^{-i\omega t} \tag{16}$$

convert the continuity and momentum equations into

$$\nabla \cdot \hat{\mathbf{u}} - k^2 \hat{\Psi} = \hat{Q} \tag{17}$$

and

$$\hat{\mathbf{u}} = -\nabla \hat{\Psi} + \frac{\nabla T}{T} \hat{\Psi} \tag{18}$$

Wave Equation

When the vector ∇T is null, equations (17) and (18) combine to form the elementary acoustic wave equation

$$\nabla^2 \hat{\Psi} + k^2 \hat{\Psi} = -\hat{Q} \tag{19}$$

When the temperature is variable, the momentum equation (18) gives an additional acoustic velocity component in the direction of the temperature gradient. The continuity equation then gives a wave equation with the temperature gradient as a variable coefficient. The heat conduction condition, equation(6), allows the wave equation to be expressed without $\nabla^2 T$.

$$\nabla^2 \hat{\Psi} - \frac{\nabla T}{T} \cdot \nabla \hat{\Psi} + \left(k^2 + \left| \frac{\nabla T}{T} \right|^2 \right) \hat{\Psi} = -\hat{Q} \tag{20}$$

The gradient of the acoustic potential is eliminated by introducing a modified potential $\hat{\Phi}$ with an integrating factor.

$$\hat{\Psi} = \sqrt{T/T_0}\, \hat{\Phi} \tag{21}$$

The resulting wave equation then depends on the magnitude of the logarithmic gradient of the temperature.

$$\nabla^2 \hat{\Phi} + \left(k^2 + \frac{1}{4} \left| \frac{\nabla T}{T} \right|^2 \right) \hat{\Phi} = -\sqrt{\frac{T_0}{T}}\, \hat{Q} \tag{22}$$

The gradient term in the wave equation can be expressed as a frequency parameter. The product of the speed of sound and the logarithmic derivative of any variable has the dimensions of frequency so that the frequency parameter is defined as

$$\omega_r = \frac{c}{2} \left| \frac{\nabla T}{T} \right| = |\nabla c| \tag{23}$$

and will be called the temperature gradient frequency. The effective frequency in the wave equation is the hypotenuse of a frequency triangle formed with the actual excitation frequency ω and the temperature gradient frequency ω_τ. The gradient frequency makes the strong gradient equation look very much like the weak gradient equation

$$\nabla^2\hat{\Phi} + \left(\frac{\omega^2 + \omega_\tau^2}{c^2}\right)\hat{\Phi} = -\sqrt{\frac{T_0}{T}}\hat{Q} \tag{24}$$

Boundary Conditions

The boundary condition for a locally-reacting surface is expressed as a proportion between the acoustic pressure and the acoustic velocity normal to the surface.

$$\hat{p} = Z\hat{u}_n \tag{25}$$

The acoustic pressure is proportional to the potential function.

$$\hat{p} = -i\omega\rho A\hat{\Phi} \tag{26}$$

The normal velocity depends on the normal gradients of both the temperature and the potential. From equations (14) and (20)

$$\hat{u}_n = -\sqrt{\frac{T}{T_0}}\left(\frac{\partial\hat{\Phi}}{\partial n} - \frac{1}{2T}\frac{\partial T}{\partial n}\hat{\Phi}\right) \tag{27}$$

The boundary condition for the potential contains the effect of the temperature gradient normal to the wall.

$$\frac{\partial\hat{\Phi}}{\partial n} = \left(\frac{1}{2T}\frac{\partial T}{\partial n} + ik\beta\right)\hat{\Phi} \tag{28}$$

The effect of the temperature gradient normal to the wall is analogous to an acoustic admittance β_τ.

$$\beta_\tau = \frac{-i}{2kT}\frac{\partial T}{\partial n} = \frac{-i}{\omega}\frac{\partial c}{\partial n} \tag{29}$$

which is purely imaginary and thus introduces no dissipation.

ACOUSTIC RESPONSE OF A RECTANGULAR CAVITY

Governing Equations

The strong-gradient wave equation will be used to predict the response of a rectangular cavity to a monopole source of strength Q_s. Place the origin of coordinates in the center of the cavity as shown in Fig. 2. With the temperature gradient in the positive z-direction, the temperature is given by

$$T = \bar{T}(1 + \tau z) \tag{30}$$

The thermal gradient frequency is a function of the gradient τ.

$$\omega_\tau = \frac{c_{\bar{T}}}{2} \frac{|\tau|}{\sqrt{1 + \tau z}} \tag{31}$$

The acoustic potential $\hat{\Phi}$ is the solution to the following wave equation

$$\nabla^2 \hat{\Phi} + \left[\frac{\omega^2 / c_{\bar{T}}^2}{1 + \tau z} + \frac{\tau^2/4}{(1 + \tau z)^2} \right] \hat{\Phi} = \frac{-\hat{Q}_s}{\sqrt{1 + \tau z_s}} \delta(x - x_s)\delta(y - y_s)\delta(z - z_s) \tag{32}$$

The solution will be developed for the case of rigid walls such that the acoustic velocity normal to the wall is zero. The velocity is a linear combination of the potential gradient and the temperature gradient vectors.

$$\hat{\mathbf{u}} = -\sqrt{1 + \tau z} \nabla \hat{\Phi} + \mathbf{k} \left(\frac{1}{2} \frac{\tau}{\sqrt{1 + \tau z}} \right) \hat{\Phi} \tag{33}$$

The boundary conditions for the acoustic potential in the rectangular cavity are homogeneous and depend on the gradient in the corresponding direction.

$$\frac{\partial \hat{\Phi}}{\partial x} = 0, \; x = \pm \frac{L}{2} \tag{34a}$$

$$\frac{\partial \hat{\Phi}}{\partial y} = 0, \; y = \pm \frac{W}{2} \tag{34b}$$

$$(1 + \tau z) \frac{\partial \hat{\Phi}}{\partial z} = \frac{\tau}{2} \hat{\Phi}, \; z = \pm \frac{H}{2} \tag{34c}$$

The wave equation (32) and boundary conditions (34) define $\hat{\Phi}$ within the rectangular cavity. The x-boundary conditions (34a) are satisfied by a set of modes $g_n(k_{xn}x)$ defined below.

$$g_n(k_{xn}x) = \sqrt{2}\cos(k_{xn}x - \theta_n), \; n = 0, 1, 2, \ldots \tag{35a}$$

$$k_{xn} = \frac{n\pi}{L} \tag{35b}$$

$$\theta_0 = \pi/4 \tag{35c}$$

$$\theta_n = \begin{cases} 0, & \text{if } n \text{ is even;} \\ \pi/2, & \text{if } n \text{ is odd.} \end{cases} \tag{35d}$$

The integral of $g_n^2(k_{xn}x)$ over the interval $(-L/2, L/2)$ is equal to the length of the interval.

$$\int_{-L/2}^{L/2} g_n(k_{xn}x) g_{n'}(k_{xn'}x)dx = \delta_{nn'}L \tag{36}$$

The potential function is expanded in a double series of modes whose amplitudes depend on the vertical position z.

$$\hat{\Phi}(x, y, z; \omega) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \hat{\Phi}_{nm}(z; \omega) g_n(k_{xn}x) g_m(k_{ym}y) \tag{37}$$

Each function $\hat{\Phi}_{nm}(z;\omega)$ is defined by an ordinary differential equation.

$$\frac{d^2\hat{\Phi}_{nm}}{dz^2} + \left[\frac{\omega^2/c_f^2}{1+\tau z} + \frac{\tau^2/4}{(1+\tau z)^2} - k_{nm}^2\right]\hat{\Phi}_{nm} = -\frac{\hat{Q}_{nm}\delta(z-z_s)}{\sqrt{1+\tau z}} \tag{38}$$

The horizontal mode numbers are

$$k_{nm}^2 = k_{xn}^2 + k_{ym}^2 \tag{39}$$

The generalized source depends on the monopole source strength, and the values of the mode functions at the source position.

$$\hat{Q}_{nm} = \frac{\hat{Q}_s}{LW} g_n(k_{xn}x_s) g_m(k_{ym}y_s) \tag{40}$$

Finally, the boundary condition (34c) carries over to each mode in the same form.

$$(1+\tau z)\frac{d\hat{\Phi}_{nm}}{dz} = \frac{\tau}{2}\hat{\Phi}_{nm}, \quad z = \pm\frac{H}{2} \tag{41}$$

A rough estimate of the natural frequency of a mode nm in a thin cavity is given by assuming

$$\frac{\omega^2}{c_f^2} + \frac{\tau^2}{4} = k_{nm}^2 \tag{42}$$

This simple estimate indicates that the natural frequency will be less than the natural frequency $\omega_{nm} = c_f k_{nm}$ of a cavity without a temperature gradient.

$$\omega_{nm} = \sqrt{\omega_{nm}^2 - \omega_{ro}^2} \tag{43}$$

The effect of the gradient-dependent boundary conditions on the acoustic modes is developed in the following section.

Basis Functions

Basis functions in the vertical dimension are generalizations of the cosine functions in the horizontal dimensions.

$$g_l(k_{zl}z) = a_l \cos(k_{zl}z - \theta_{zl}) \tag{44}$$

These functions must satisfy boundary conditions which depend on the temperature gradient.

$$(1 + \tau z)g_l'(k_{zl}z - \theta_{zl}) = \frac{\tau}{2} g_l(k_{zl}z - \theta_{zl}), \quad z = \pm\frac{H}{2} \tag{45}$$

The boundary conditions yield the following characteristic equation for the wave number k_{zl}.

$$\frac{\tan k_{zl}H}{k_{zl}H} = \frac{2\epsilon^2}{(4 - \epsilon^2)k_{zl}^2H^2 + \epsilon^2} \tag{46}$$

$$\epsilon = \tau H = \Delta T/\bar{T} \tag{47}$$

The series for $\tan x/x$ contains only even powers so that $(k_{zl}H)^2$ is a function of $(\Delta T/\bar{T})^2$. The branch is selected such that the two variables have the same sign when they are real-valued. The series also provide asymptotic formulas for the wave numbers.

$$k_{z0}H = \frac{\epsilon}{2}\left(1 + \frac{1}{24}\epsilon^2 + \cdots\right) \tag{48a}$$

$$k_{zl}H = l\pi\left(1 + \frac{1}{2l^2\pi^2}\epsilon^2 + \cdots\right) \tag{48b}$$

The lowest order wave number is proportional to the temperature gradient, but the higher-order wave numbers are functions of the square of the temperature gradient.

The phase of the modes is found from either boundary condition.

$$\tan\theta_{zl} = \pm\frac{(2 \pm \epsilon)k_{zl}H\sin k_{zl}H/2 \pm \epsilon\cos k_{zl}H/2}{(2 \pm \epsilon)k_{zl}H\cos k_{zl}H/2 \mp \epsilon\sin k_{zl}H/2} \tag{49}$$

The dominant term in the asymptotic formula for the phase of the modes shows that the modes approach the simpler sine and cosine functions of the zero-gradient case.

$$\theta_{x0} = \frac{\pi}{4} + \frac{1}{24}\epsilon^2 + \cdots \tag{50a}$$

$$\theta_{xl} = \left\{ \begin{array}{l} 0,\ l\ even \\ \pi/2,\ l\ odd \end{array} \right\} + \frac{\epsilon}{2l\pi} \tag{50b}$$

Thus the effect of the temperature gradient is to give the fundamental mode a small wave number which is proportional to the gradient. The fundamental mode becomes slightly nonuniform, varying slowly with distance, because k_{x0} is proportional to ϵ. The higher modes have wave numbers shifted by the square of the gradient, but the phase of the mode is shifted in proportion to the gradient. The principal effect is to translate the mode in the positive or negative direction, depending on the sign of the gradient, as shown by equation (50b). The modes are neither even nor odd functions, but have a mixed character because of the gradient.

The normalization constant a_l depends on the wave number and phase of the mode.

$$a_l = \sqrt{2}\left(1 + \frac{sin\ k_{xl}H}{k_{xl}H} cos\ 2\theta_l\right)^{-1/2} \tag{51}$$

The expansion of equation (51) shows that the effect of the gradient on the normalization constant is of order ϵ^2.

$$a_0 = \sqrt{2}\left(1 + \frac{1}{24}\epsilon^2 \cdots\right) \tag{52a}$$

$$a_l = \sqrt{2}\left(1 - \frac{1}{4l^2\pi^2}\epsilon^2 \cdots\right) \tag{52b}$$

Galerkin Solution

An approximate solution to the modal wave equations (38) and (41) can be made by extending the mode expansion to the vertical direction and selecting the mode amplitudes by the Galerkin criterion[9]. The result is a set of algebraic equations which are coupled in the vertical index but uncoupled in the horizontal indices.

$$\sum_{l'=0}^{\infty}\left[\frac{\omega^2}{c_{\bar{T}}^2}A_{ll'} + \frac{\tau^2}{4}B_{ll'}\right]\hat{\Phi}_{nml'} - \left(k_{nm}^2 + k_{xl}^2\right)\hat{\Phi}_{nml} = -\hat{Q}_{nml} \tag{53}$$

$$A_{ll'} = \frac{1}{H}\int_{-H/2}^{H/2} \frac{g_l(k_{zl}z)g_{l'}(k_{zl'}z)}{(1+\tau z)}dz \tag{54}$$

$$B_{ll'} = \frac{1}{H}\int_{-H/2}^{H/2} \frac{g_l(k_{zl}z)g_{l'}(k_{zl'}z)}{(1+\tau z)^2}dz \tag{55}$$

The integrals in equations (54) and (55) are expressible as sine and cosine integrals. Alternatively, power series may be used to develop approximations for the effect of the temperature gradient. Second-order- accurate approximations will be used here to clarify the effect of the temperature gradient on the modal functions and cavity response. Since $B_{ll'}$ is multiplied by $\tau^2/4$ in equation (53), only the zero-order approximation, $\delta_{ll'}$ is needed for these integrals. The second-order response equation is thus

$$\left(\bar{\omega}_{nml}^2 + \epsilon^2 \frac{3}{4\pi^2} u_l \bar{\omega}_z^2\right)\hat{\Phi}_l - \omega^2 \sum_{l'=0}^{\infty} A_{ll'}\hat{\Phi}_{l'} = c_{\bar{T}}^2 \hat{Q}_l \tag{56}$$

where

$$u_l = \begin{cases} 0, & l = 0 \\ 1, & l > 0 \end{cases} \tag{57}$$

and $A_{ll'}$ is approximated by the first three terms of a power series in ϵ

$$A_{ll'} = \delta_{ll'} + \epsilon A'_{ll'} + \frac{\epsilon^2}{2} A''_{ll'} \tag{58}$$

The frequencies $\bar{\omega}_{nml}$ are the resonant freqencies of a cavity with constant temperature $\bar{T}$

$$\bar{\omega}_{xn} = c_{\bar{T}} \frac{n\pi}{L} \tag{59a}$$

$$\bar{\omega}_{ym} = c_{\bar{T}} \frac{m\pi}{W} \tag{59b}$$

$$\bar{\omega}_{lz} = c_{\bar{T}} \frac{l\pi}{H} \tag{59c}$$

$$\bar{\omega}_{nml}^2 = \bar{\omega}_{xn}^2 + \bar{\omega}_{ym}^2 + \bar{\omega}_{zl}^2 \tag{59d}$$

It is understood that $\bar{\omega}_z = \bar{\omega}_{z1}$. In the following equations, the subscript (n,m) will be suppressed for brevity. When $\bar{\omega}_l$ appears, it means ω_{nml}. Equivalent interpretations are to be used for the symbols $\hat{\Phi}_l$ and $\hat{Q}_l$.

The modes and their integrals will be given terms of a dimensionless coordinate $\xi = z/H$ which has the range (-1/2, 1/2). The second-order approximations for the modes are

$$g_0(k_{z0}H\xi) \approx 1 + \epsilon\frac{\xi}{2} - \epsilon^2\frac{\xi^2}{8} \tag{60a}$$

$$g_l(k_{zl}H\xi) \approx \sqrt{2}\sin l\pi\xi - \epsilon\frac{\sqrt{2}}{2l\pi}\cos l\pi\xi + \epsilon^2\left(\frac{\sqrt{2}}{2l\pi}\xi\cos l\pi\xi - \frac{3\sqrt{2}}{8\pi^2 l^2}\sin l\pi\xi\right), l = 1,3,5,... \tag{60b}$$

$$g_l(k_{zl}H\xi) \approx \sqrt{2}\cos l\pi\xi + \epsilon\frac{\sqrt{2}}{2\pi l}\cos l\pi\xi - \epsilon^2\left(\frac{\sqrt{2}}{2\pi l}\xi\sin l\pi\xi + \frac{3\sqrt{2}}{8\pi^2 l^2}\cos l\pi\xi\right), l = 2,4,6... \tag{60c}$$

Equations (60) shows the effect of the temperature gradient on the acoustic modes. The modes are no longer even and odd functions in correspondence with their index. Each mode has an odd (in ξ) part, which is proportional to the temperature gradient, and each even-numbered mode has an odd part. The appearance of the functions $\xi sin(l\pi\xi)$ and $\xi cos(l\pi\xi)$ in the second-order gradient effects indicates that a Fourier series approximation to the strong-gradient solution would be accurate to only first order in gradient.

The first derivative $A'_{ll'}$ are found to be zero unless $l + l'$ is an odd number.

$$A'_{ll'} = 0, \quad l + l' = 0,2,4... \tag{61a}$$

$$A'_{l0} = (-1)^{(l+1)/2}\frac{2\sqrt{2}}{l^2\pi^2}, \quad l = 1,3,5... \tag{61b}$$

$$A'_{ll'} = (-1)^{(l+l'+1)/2}\frac{4}{\pi^2}\frac{\left(l^2 + l'^2\right)}{\left(l^2 - l'^2\right)^2}, \quad \begin{cases} l + l' = 1,3,5... \\ l > 0, \quad l' > 0 \end{cases} \tag{61c}$$

The second derivatives are zero unless $l + l'$ is an even number

$$\frac{1}{2}A''_{00} = 0 \tag{61d}$$

$$\frac{1}{2}A''_{l0} = (-1)^{l/2}\frac{3\sqrt{2}}{2l^2\pi^2}, \quad l = 2,4,6... \tag{61e}$$

$$\frac{1}{2}A''_{ll'} = \left(\frac{l}{12} + \frac{1}{l^2\pi^2}\right) \quad l = 1,2,3... \tag{61f}$$

$$\frac{1}{2}A''_{ll'} = (-1)^{(l+l')/2}\frac{4}{\pi^2}\frac{\left(l^2 + l'^2\right)}{\left(l^2 - l'^2\right)^2}, \quad \begin{cases} l + l' = 2,4,6... \\ l > 0, \quad l' > 0 \end{cases} \tag{61g}$$

Natural Frequencies and Modes

Homogeneous solutions of equations (56) are possible at resonant frequencies ω_l which depend on the temperature gradient parameter ϵ. These resonant frequencies are solutions of the infinite determinant equation

$$\left|\omega^2 A_{ll'} - \delta_{ll'}\left(\bar{\omega}_l^2 + \epsilon^2 s_l \bar{\omega}_z^2\right)\right| = 0 \tag{62}$$

When this determinant is expanded, it is found that the cofactor of any element of order ϵ is also of order ϵ. Consequently, the resonant frequencies are functions of ϵ^2 and can be approximated as

$$\omega_l^2 \approx \bar{\omega}_l^2\left(1 + \epsilon^2 \alpha_l\right) \tag{63}$$

The corresponding natural modes, designated by x_{li} are given as

$$x_{li} = x_{li}^{(0)} + \epsilon x_{li}^{(1)} + \epsilon^2 x_{li}^{(2)} \tag{64}$$

Since the homogeneous solution (64) may be multiplied by an arbitrary constant, there is no loss of generality in taking $x_{ll}^{(0)} = 1$, $x_{ll}^{(1)} = 0$ and $x_{ll}^{(2)} = 0$. The remaining elements of the vectors are found by substituting the assumed solution (64) with the series (58) for $A_{ll'}$ into the homogeneous equation from (56) and collecting coefficients of each power of ϵ. The zero order vectors are the Kronecker delta function

$$x_{ll'}^{(0)} = \delta_{ll'} \tag{65}$$

The first order vectors are proportional to the first derivatives $A_{ll'}$

$$x_{ll'}^{(1)} = \mathcal{D}_l^2 \begin{cases} 0, & l = l' \\ \frac{A_{ll'}'}{l'^2 - l^2}, & l \neq l' \end{cases} \tag{66}$$

where $\mathcal{D}_l$ is the reduced frequency for the mode

$$\mathcal{D}_l^2 = \left(\frac{\bar{\omega}_l}{\bar{\omega}_z}\right)^2 = \left(\frac{nH}{L}\right)^2 + \left(\frac{mH}{W}\right)^2 + l^2 \tag{67}$$

The second-order vectors are found to depend on the second derivative $A_{ll'}''$ and on products of the first derivatives.

$$x_{ll'}^{(2)} = \mathcal{D}_l^2\left[\frac{1}{2}\frac{A_{ll'}''}{l^2 - l'^2} + \left(\frac{2}{\pi}\right)^4 \mathcal{D}_l^2 C_{ll'}\right], \quad l \neq l' \tag{68}$$

where

$$C_{ll'} = \left(\frac{\pi}{2}\right)^4 \sum_{\substack{j=0 \\ j \neq l'}}^{\infty} \frac{A'_{jl} A'_{jl'}}{(l'^2 - j^2)} \tag{69}$$

The frequency-shift constant is found to be a function of ν_l^2 from the second order equation.

$$\alpha_l = \left(\frac{2}{\pi}\right)^4 C_{ll} \nu_l^2 - \frac{1}{2} A''_{ll'} + \frac{3}{4\pi^2} \frac{u_l}{\nu_l^2}, \quad \nu_l > 0 \tag{70}$$

Returning the subscripts n and m, the final equation is given for the natural frequencies of a rectangular cavity with a strong temperature gradient. These equations are the principle result of this paper

$$\omega_{nml}^2 = \bar{\omega}_{nml}^2 \left[1 + \alpha_{nml} \left(\frac{\Delta T}{\bar{T}}\right)^2\right] \tag{71a}$$

$$\alpha_{nm0} = \left(\frac{2}{\pi}\right)^4 C_{00} \frac{\bar{\omega}_{nm0}^2}{\bar{\omega}_{001}^2} \tag{71b}$$

$$\alpha_{nml} = \left(\frac{2}{\pi}\right)^4 C_{ll} \frac{\bar{\omega}_{nml}^2}{\bar{\omega}_{001}^2} - \left(\frac{1}{12} + \frac{1}{l^2\pi^2}\right) + \frac{3}{4\pi^2} \frac{\bar{\omega}_{001}^2}{\bar{\omega}_{nml}^2}, \quad l > 0 \tag{71c}$$

The constant C_{00} is negative, roughly -1/2. Consequently, the effect of the temperature gradient is to shift all resonant frequencies ω_{mn0} to lower values. The change in the square of the frequency is proportional to the square of the temperature gradient. For all higher modes, $l \geq 0$, the resonant frequencies may shift in either direction. The shift of frequency-squared is again proportional to gradient-squared, but the constant of proportionality is a quadratic function of the square of the reduced frequency for the mode. This reduced frequency is equal to or greater than the mode index l and, in the case of equality, it can be shown that the quadratic function has a value less than zero so that resonant frequencies shift to lower values. The quadratic function is greater than zero for large reduced frequencies because the constant C_{ll} is positive. There is a value of the reduced frequency ν_l^*, the greater root of the quadratic, which is greater than the index l. Each mode whose reduced frequency is in the range $[l, \nu_l^*)$ will shift to lower resonant frequencies with increasing temperature gradient, and each mode with reduced frequency in the range (ν_l^*, ∞) will shift to higher frequency.

ACOUSTIC WAVEGUIDE EXPERIMENT

A rectangular channel with a one-dimensional temperature distribution can be constructed as shown in Fig. 3 to investigate the effect of strong temperature gradient on acoustic modes. The width is made small compared to the height so that cross modes, $m > 0$, respond only at high frequencies. It is necessary to cool the lower surface while heating the upper in order to keep the temperature extremes within the range of available acoustic pressure sensors. A set of five temperature combinations is given in Table I which gives evenly-spaced values of $(\Delta T/\bar{T})^2$ from zero to 1. The average temperature is held fixed at the boiling point of water, 393.15 K. With this choice, the lowest temperature is 196.6 K, slightly higher than the sublimation point of dry ice (CO_2) which is 194.4 K. The highest temperature is 589.7 K. The height of the channel is chosen to give a convenient reference frequency of 1 kHz. Based on the speed of sound at 393.15 K, which is 397.49 m/sec, the height is selected to be 19.87 cm. The length is selected to be a nonintegral multiple of height, 5/2, to avoid low-frequency duplication of longitudinal and vertical mode resonant frequencies. The length is set at 49.69 cm. These dimensions give temperature gradients up to 20 K/cm, roughly the same size as those estimated for the thermal protection system panel in Fig 1.

Table II shows resonant frequencies up to 4 kHz for this cavity. The corresponding frequency-shift constants are shown in Table III. It is clear from Table III that the largest effect of the temperature gradient is on the plane wave (n,0,0) modes and the first antisymmetric (n,0,1) modes. Fig. 4 shows two examples of these modes, the (5,0,0) and (5,0,1) modes.

If the acoustic drivers in Fig. 3 are operated in phase, only the plane wave modes will be excited directly. The antisymmetric modes will not be excited because the excitation is symmetric. The drivers are located at $z = \pm H/4$ which are the node lines of the (n,0,2) and higher order symmetric modes. These positions eliminate the higher order symmetric modes. Of course, all modes will receive some excitation as the temperature gradient is increased because the modes are no longer perfectly symmetric or antisymmetric and the node lines will be shifted. The drivers may be operated 180 degrees out of phase to excite the antisymmetric modes with only small excitation of the symmetric modes.

Resonant frequencies are given in Tables IV and V for the plane-wave and first an-

tisymmetric modes, respectively. The (10,0,0) mode has imaginary frequencies in the extreme case where $\Delta T/\bar{T} = 1$, but this is outside the limit where the asymptotic theory is valid. In any event, the tendency of increasing gradient should be to decrease the resonant frequencies of the plane-wave modes so the waveguide response should be as shown in Fig. 5 with symmetric excitation. With no temperature gradient, there should be 6 resonant peaks between 0 and 2.5 kHz. When the temperature gradient is increased to $\Delta T/\bar{T} = 1/2$, the lower resonant peaks should be unaffected, but there should be a perceptible downshift of the 4th, 5th and 6th peaks. When the temperature gradient is increased to $\sqrt{2}/2$ there should be a more pronounced shift and a 7th peak should appear.

Fig. 6 shows the anticipated response of a waveguide to antisymmetric excitation. The response is shown from 2 to 4.4 kHz because the resonant frequencies should shift upward in this range. Without a temperature gradient, ther are five resonant peaks, the lowest at 2236 Hz and the highest at 3736 Hz. As the temperature gradient is increased to $\sqrt{2}/2$, the lowest peak should shift upward by about 100 Hz and the highest should shift up by over 600 Hz.

CONCLUDING REMARKS

Temperature gradients is gases affect acoustic waves by way of a spatially-variable sound speed. In general, the sound speed and its derivatives appear in the wave equation in a function which is a variable coefficient of the acoustic potential. When the gradients are small, the derivatives can be neglected and the function depends only on the speed of sound. This approximation has been called the weak gradient theory in this paper. There are problems where the gradients are not small, (on the scale of a typical wavelength) and a modified wave equation has been developed here for these cases. This modified equation has been called a strong gradient wave equation. The essential modification is the addition of the square of the sound speed gradient to the square of the circular frequency. The acoustic boundary conditions are altered by adding a negative imaginary admittance β_τ to the wall. The magnitude of this admittance is the ratio of the normal derivative of the sound speed to the circular frequency.

The strong gradient theory has been applied to determine the resonant frequencies of rectangular cavities with temperature gradients in the direction of a one coordinate. It has been shown that the resonant frequencies of the cavity are changed in proportion to the square of the temperature gradient. Asymptotic formulas, accurate to second order in the temperature gradient, have been developed for the resonant frequencies and normal modes of the cavity.

These formulas show that the modes with small variation in the gradient direction will all shift to lower resonant frequencies. Higher modes, both symmetric and antisymmetric in the gradient direction, may shift to lower or to higher resonant frequencies, depending on their zero gradient frequency.

A conceptual experiment has been defined to study the effects of gradients on resonant frequencies. The experiment uses a two-dimensional waveguide roughly 20 cm high by 50 cm long to study the frequency range from 400 Hz to 4 kHz. The average temperature in the waveguide is held fixed at about the boiling point of water, 393 K while varying the temperature of the upper and lower surfaces of the waveguide. Heating the upper surface while cooling the lower produces a thermally-stable stratification of air with a nearly-constant temperature gradient. Symmetric excitation of the waveguide should

excite longitudinal, nearly-plane, modes whose resonant frequencies will shift downward with increasing temperature gradient. Antisymmetric excitation may be used to excite longitudinal modes whose frequencies should shift upward with increasing gradient. This experiment will provide a critical test of the strong gradient theory.

REFERENCES

1. Pekeris, C. L.: Theory of Propagation of Sound in a Half-Space of Variable Sound Velocity under Conditions of Formation of a Shadow Zone. J. Acoust. Soc. Amer., Vol. 18, No. 2, October, 1946. pp. 295–315.

2. Brekhovskikh, L. M.: Waves in Layered Media. Second Edition Translated by Robert T. Beyer. Academic Press, 1980.

3. Pridmore-Brown, David C.: Sound Propagation in a Temperature- and Wind-Stratified Medium. J. Acoust. Soc. Amer. Vol. 34, No. 4, April 1962. pp. 438–443.

4. Grossard, Earl E.; and Hooke, William H.: Waves in the Atmosphere. Atmospheric Infrasound and Gravity Waves — their Generation and Propagation. Elsevier Scientific Publishing Company, 1975.

5. Blair, Winford: Manufacturing Experiences for Advanced Multiwall Prepackaged Metallic TPS. *Advances in TPS and Structures for Space Transportation Systems.* NASA Conference Publication 2315, Decenber 1983. pp. 261–302.

6. Attenborough, Keith: Acoustic Characteristics of Rigid Absorbents and Granular Materials. J. Acoust. Soc. Amer., vol. 73, No. 3, March 1983. pp. 785–799.

7. Tijdeman, H.: Investigations of the transonic flow around oscillating airfoils. NLR-TR-77090-U, Oct. 21, 1977.

8. Tijdeman, H.: On the Propagation of Sound Waves in Cylindrical Tubes. Journal of Sound and Vibration, vol. 39, 1975, pp. 1–33

9. Finlayson, Bruce A.: The Method of Weighted Residuals and Variational Principles with Application in Fluid Mechanics, Heat and Mass Transfer. Academic Press, Inc. 1972.

10. Gradshteyn, I. S.; and Ryzhik, I. M.: Table of Intrgrals, Series, and Products. Translation edited by Alan Jeffrey. Academic Press, 1965.

TABLE I

Waveguide Experiment Temperatures

L=49.14 cm, H=19.66cm, T=393.15 K					
$\Delta T/\bar{T}$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
T_1	393.2	294.9	254.2	229.9	196.6
T_2	393.2	491.4	532.1	563.4	589.7
$\Delta T, K$	0	196.6	278.0	340.5	393.2
$T', K/cm$	0	11.80	14.14	17.32	20.00

TABLE II

Waveguide Natural Frequencies

L=49.69 cm, H=19.87cm, $\bar{T}$=393.15 K					
n/l	0	1	2	3	4
0	0	1000	2000	3000	4000
1	400	1077	2040	3027	
2	800	1281	2154	3105	
3	1200	1562	2332	3231	
4	1600	1887	2561	3400	
5	2000	2236	2828	3606	
6	2400	2600	3124	3842	
7	2800	2973	3441		
8	3200	3353	3774		
9	3600	3736			
10	4000				

TABLE III

Frequency Shift Constants

l	0	1	2	3	4
C_{ll}	-5.007(-1)	3.97(-1)	4.88(-2)	1.74(-2)	8.89(-3)
n	$\alpha_{n0l} \times 100$				
0	0	-4.35	-5.76	-6.04	-6.13
1	-1.32	-4.36	-5.71	-6.01	
2	-5.26	-3.14	-5.51	-5.91	
3	-11.8	0.55	-5.11	-5.74	
4	-21.1	6.9	-4.45	-5.49	
5	-32.9	15.6	-3.51	-5.15	
6	-47.4	26.7	-2.26	-4.73	
7	-64.5	40.0	-0.73		
8	-84.2	54.7	1.08		
9	-106.6	101.9			
10	-131.6				

TABLE IV

Plane Wave Mode Resonance

f_{n00}, Hz

$\Delta T/T$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
1	400	399	399	398	397
2	800	797	794	791	788
3	1200	1182	1164	1146	1127
4	1600	1557	1513	1468	1421
5	2000	1916	1828	1736	1638
6	2400	2253	2096	1927	1741
7	2800	2564	2305	2012	1668
8	3200	2843	2435	1943	1272
9	3600	3083	2460	1612	925
10	4000	3277	2330	456	2449

TABLE V

First Antisymmetric Mode Resonance

f_{n01}, Hz

$\Delta T/\bar{T}$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
0	1000	978	956	932	909
1	1077	1071	1065	1059	1053
2	1281	1276	1271	1266	1261
3	1562	1563	1564	1565	1566
4	1887	1889	1890	1892	1893
5	2236	2279	2322	2363	2404
6	2600	2685	2768	2848	2927
7	2973	3118	3257	3390	3518
8	3353	3575	3784	3982	4170
9	3736	4063	4365	4648	4914

APPENDIX A

Evaluation of Constants C_{ll}

The series for C_{ll} converge rapidly. Since the derivatives A'_{lj} are proportional to $(l-j)^{-2}$, the terms in the series are proportional to $(l-j)^{-5}$. The series for C_{ll} can be arranged as the difference between a sum and a series, both of which are positive.

$$C_{ll} = C_{ll}^{>} - C_{ll}^{<} \tag{A1}$$

$$C_{11}^{>} = 1/2 \tag{A2}$$

$$C_{ll}^{>} = \frac{1}{2l^6} + \sum_{j=l-1,l-3,\ldots}^{2} \frac{\left(l^2+j^2\right)^2}{\left(l^2-j^2\right)^5}, \quad l = 3,5,\ldots \tag{A3}$$

$$C_{ll}^{<} = \sum_{j=l+1,l+3,\ldots}^{\infty} \frac{\left(l^2+j^2\right)^2}{\left(l^2-j^2\right)^5}, \quad l = 1,3,5,\ldots \tag{A4}$$

$$C_{ll}^{>} = \sum_{j=l-1,l-3,\ldots}^{1} \frac{\left(l^2+j^2\right)^2}{\left(l^2-j^2\right)^5}, \quad l = 2,4,\ldots \tag{A5}$$

$$C_{ll}^{<} = \sum_{j=l+1,l+3,\ldots}^{\infty} \frac{\left(l^2+j^2\right)^2}{\left(l^2-j^2\right)^5}, \quad l = 2,4,\ldots \tag{A6}$$

An error of about 3^{-5}, (less than 1%) results from taking a single term in each series, so that approximate values of C_{ll} are

$$C_{ll}^{>} = 1/2 \tag{A7}$$

$$C_{ll}^{>} \approx \frac{1}{2l^6} + \frac{\left(2l^2-2l+1\right)^2}{(2l-1)^5}, \quad l = 3,5\ldots \tag{A8}$$

$$C_{ll}^{<} \approx \frac{\left(2l^2+2l+1\right)^2}{(2l+1)^5}, \quad l = 1,3,5\ldots \tag{A9}$$

$$C_{ll}^{>} \approx \frac{\left(2l^2-2l+1\right)^2}{(2l-1)^5}. \quad l = 2,4\ldots \tag{A10}$$

$$C_{ll}^{<} \approx \frac{\left(2l^2+2l+1\right)^2}{(2l+1)^5}, \quad l = 2,4\ldots \tag{A11}$$

The difference of the positive and negative terms in the series is approximated for $l \geq 2$ by

$$\frac{\left(2l^2-2l+1\right)^2}{(2l-1)^5}-\frac{\left(2l^2+2l+1\right)^2}{(2l-1)^5} \approx \frac{1}{8l^2}\left(1-\frac{3}{4l^2}-\frac{11}{4l^4}\right), \quad l \geq 2 \qquad (A12)$$

The approximate values for C_{ll} can now be given as simple functions of l

$$C_{11} \approx \frac{193}{486} \qquad (A13)$$

$$C_{ll} \approx \frac{1}{8l^2}\left(1-\frac{3}{4l^2}-\frac{11}{4l^4}\right), \quad l = 2,4,6... \qquad (A14)$$

$$C_{ll} \approx \frac{1}{8l^2}\left(1-\frac{3}{4l^2}+\frac{5}{4l^4}\right), \quad l = 3,5,7... \qquad (A15)$$

ORIGINAL PAGE
BLACK AND WHITE PHOTOGRAPH

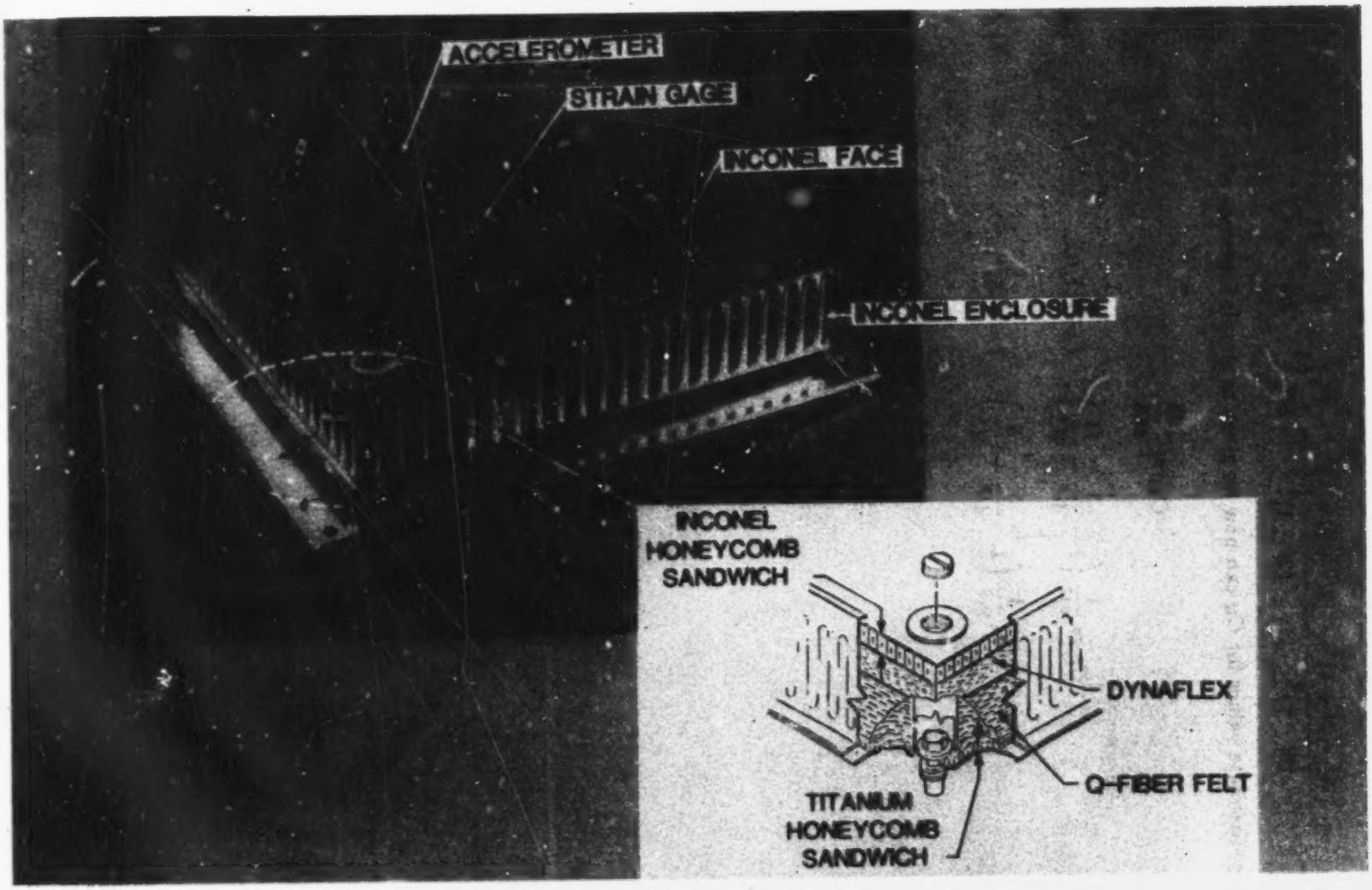


Figure 1. Bimetallic silica sandwich panel for hypersonic aircraft

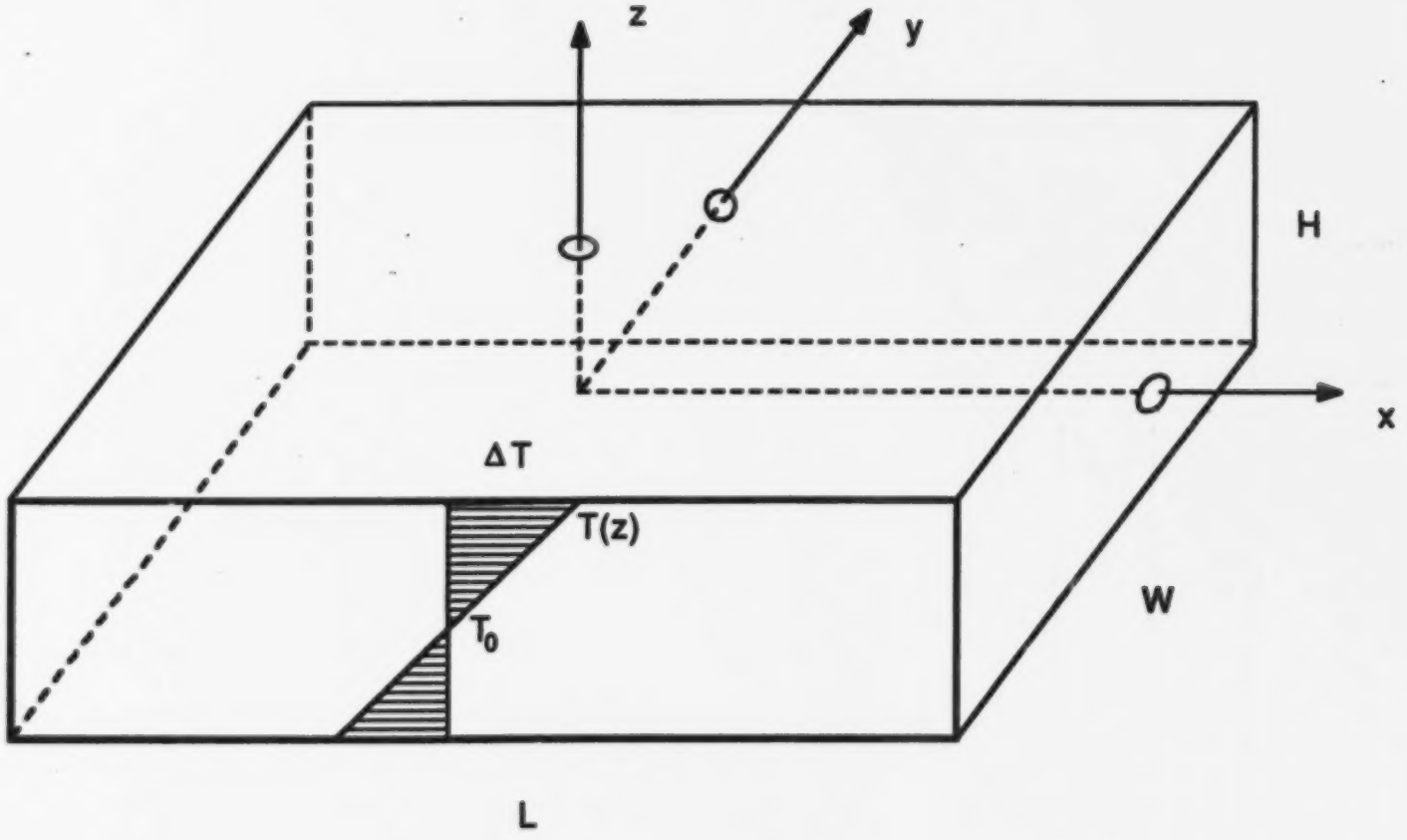


Figure 2. Rectangular cavity with a strong temperature gradient

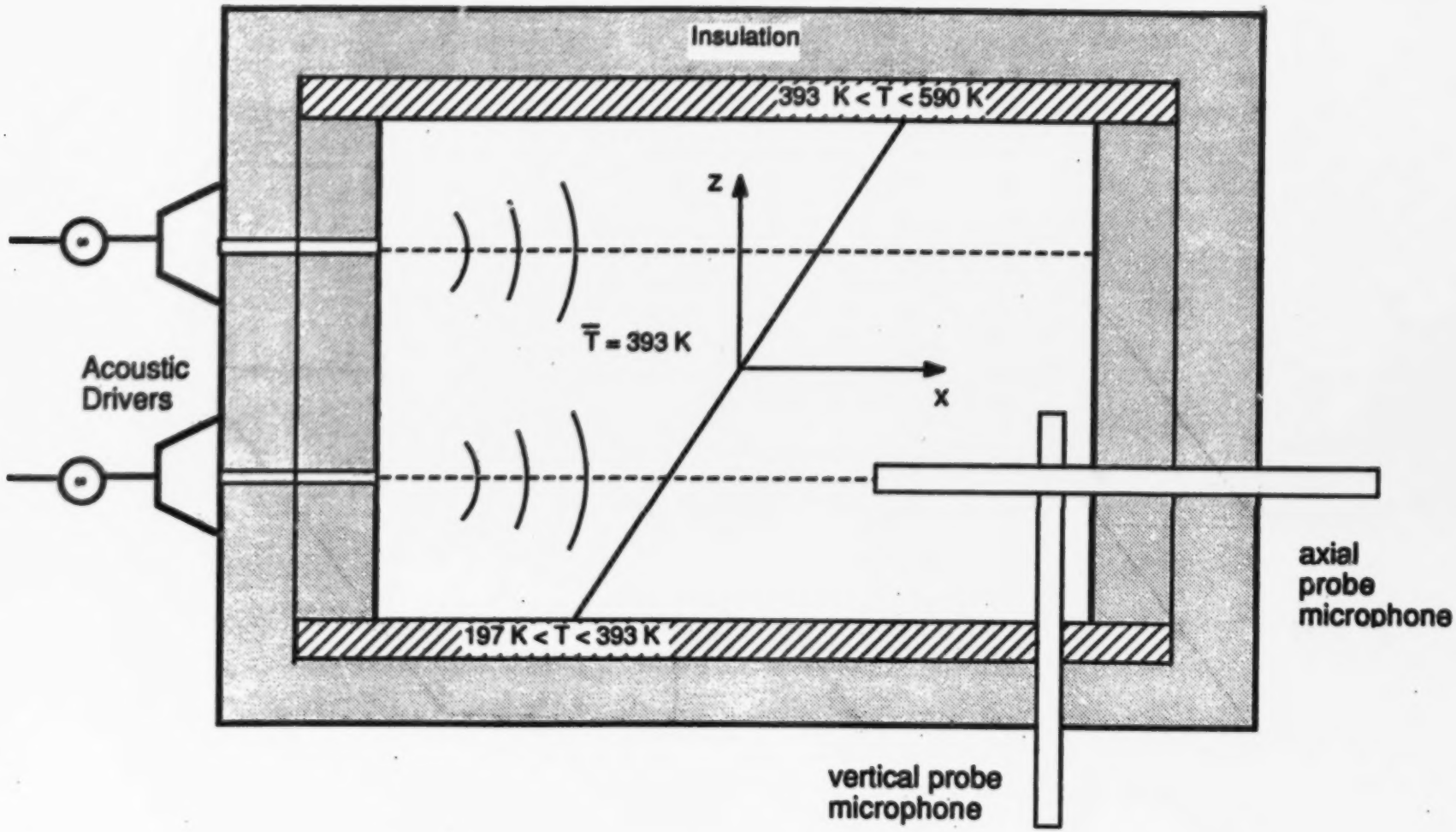
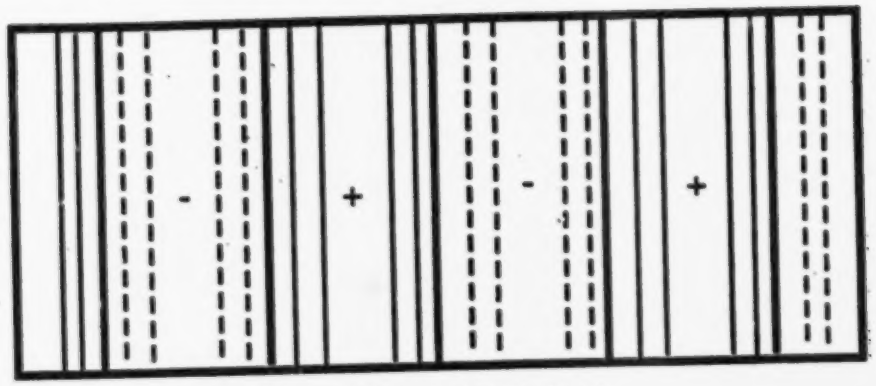
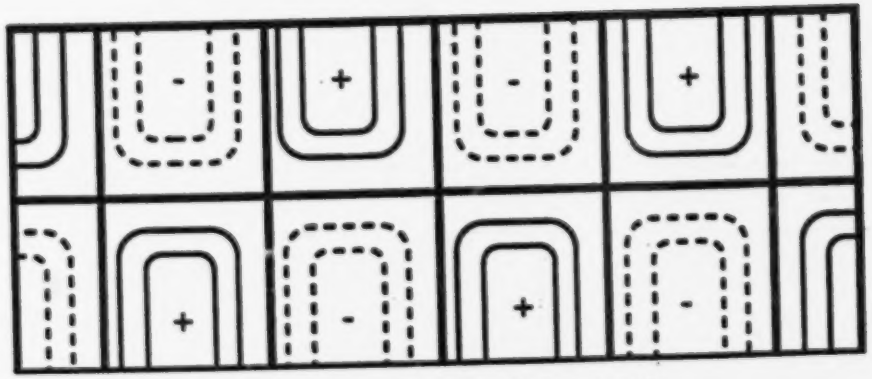


Figure 3. Acoustic waveguide apparatus for validating strong-gradient theory



Symetric mode (5, 0, 0)



Antisymetric mode (5, 0, 1)

Figure 4. Acoustic waveguide modes

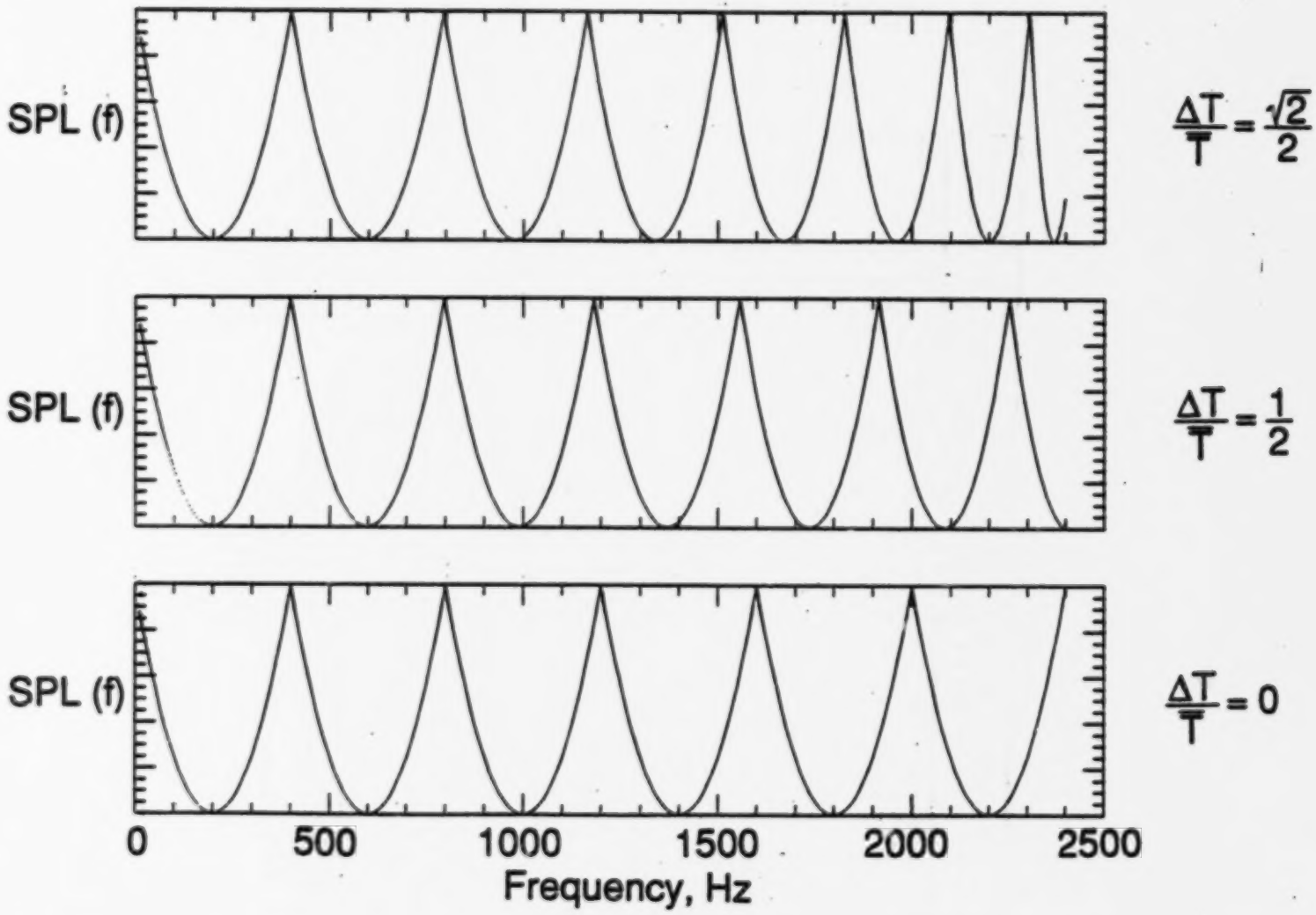


Figure 5. Waveguide response to symetric excitation

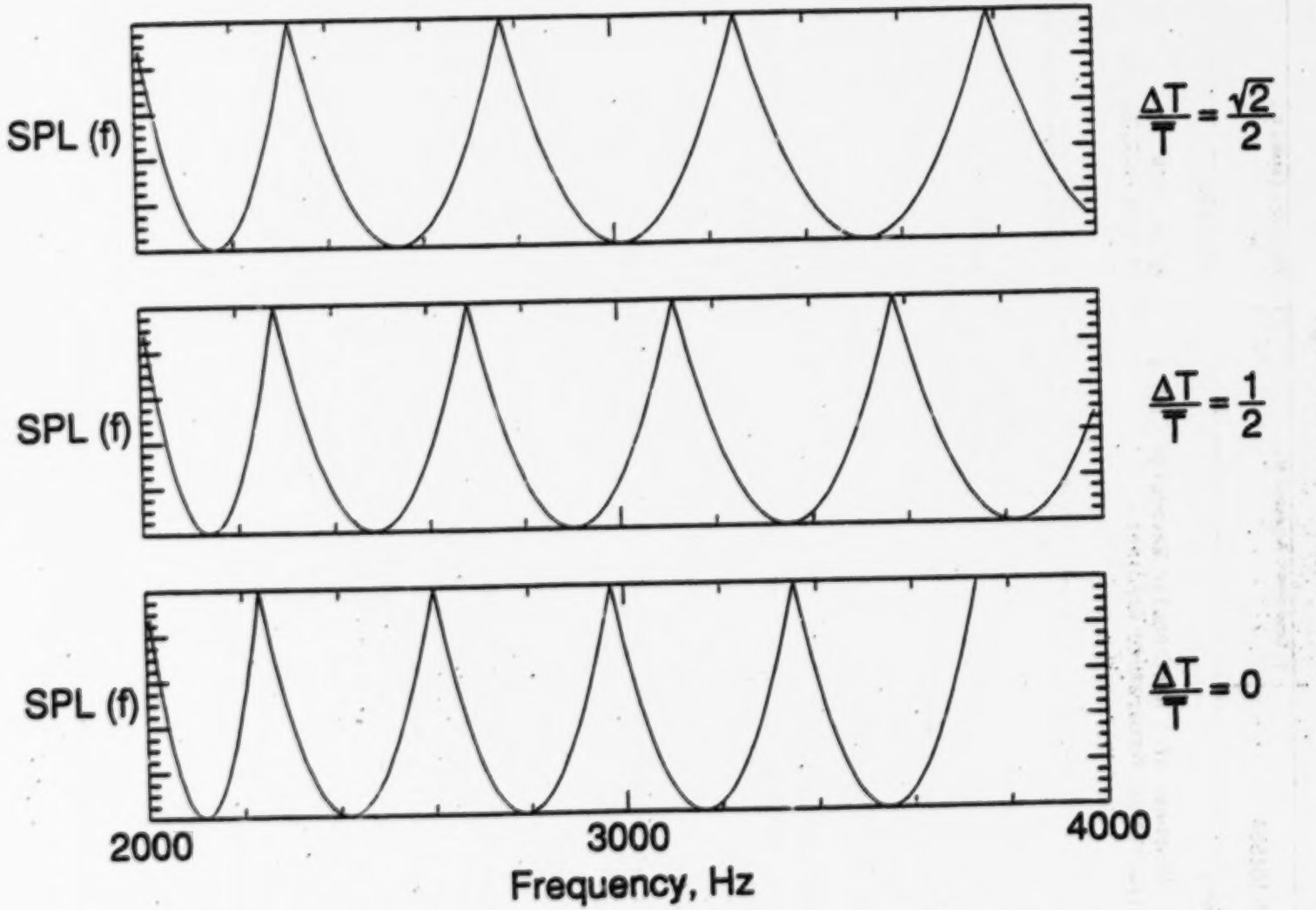


Figure 6. Waveguide response to antisymetric excitation

NASA National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-101564		

4. Title and Subtitle

Acoustic Response of a Rectangular Waveguide with a Strong Transverse Temperature Gradient

5. Report Date

March 1989

6. Performing Organization Code

7. Author(s)

William E. Zorumski

8. Performing Organization Report No.

10. Work Unit No.

505-61-11-01

9. Performing Organization Name and Address

Langley Research Center
Hampton, Virginia 23665-5225

11. Contract or Grant No.

13. Type of Report and Period Covered

Technical Memorandum

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, DC 20546-0001

14. Sponsoring Agency Code

15. Supplementary Notes

Presented at the AIAA 27th Aerospace Sciences Meeting in Reno, Nevada January 9-12, 1989.

16. Abstract

An acoustic wave equation is developed for a perfect gas with spatially-variable temperature. The strong-gradient wave equation is used to analyze the response of a rectangular waveguide containing a thermally-stratified gas. It is assumed that the temperature gradient is constant, representing one-dimensional heat transfer with a constant coefficient of conductivity. The analysis of the waveguide shows that the resonant frequencies of the waveguide are shifted away from the values that would be expected from the average temperature of the waveguide. For small gradients, the frequency shift is proportional to the square of the gradient. The factor of proportionality is a quadratic function of the natural frequency of the waveguide with uniform temperature. An experiment is designed to verify the essential features of the strong-gradient theory.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Acoustic Waves Sound Speed Gradient Temperature Gradient Acoustic Resonance	Unclassified - Unlimited Subject Category 71

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	34	A03